MA 307 META-SYLLABUS

DAN DUGGER, HAYDEN HARKER, DEV SINHA, AND ARKADY VAINTROB

1. Desired outcomes

Faculty can teach MA 307 with a wide range of course structure and content. Regardless of structure, students need to come out of the course passably fluent in mathematical language and process. While these outcomes are often not explicitly emphasized, they should be the outgrowth of a student's successful work in the class.

1.1. **Students should know how to interpret and employ "if-then" statements.** They should be able to understand and employ such statements both as proof statements and in the course of proofs. Students should be able to distinguish between a statement and its converse, and employ the contrapositive. (Such terminology isn't a necessity.)

1.2. **Students should know what counterexamples are.** They should be able to generate examples illustrating a general statement and say whether an example falsifies the statement. They should know that counterexamples are often what is required to disprove a statement. They should be able to be systematic about finding counterexamples by employing the contrapositive.

1.3. **Students should know what definitions, theorems, and proofs are.** They should know that one reads these in different "modes." For example, the logical statements in proofs should be read critically, while definitions are to be accepted by fiat. Students should understand how each of these kinds of statements pertains to examples.

1.4. **Students should be fluent with simple induction proofs.** They should be able to identify base cases and induction steps, to understand that induction proofs involve labeled statements (not equations), to be familiar with strong induction and with cases in which base cases are more complicated, and to employ induction arguments without falling into circular reasoning.

1.5. **Students should be able to interpret quantifiers.** They should be able to identify the different roles that variables play. They should be able to negate statements with quantifiers. They should be beginning to employ quantifiers appropriately in their own arguments.

2. Course content and structure

While there are a number of possible ways in which the course can be run, and pretty much any content could be justified, experience suggests some constraints on these choices.

2.1. **Practice and feedback are essential.** Precise mathematical language is generally foreign, in some cases completely foreign, to these students. They will try to mimic you, the text, and other students, but invariably they will make many mistakes. It will take some time before they naturally connect abstract statements to relevant examples, so their own ability to catch mistakes is severely limited. The more frequently these mistakes can be corrected, the more the students can progress.

2.2. **The mathematics needs to be focussed somehow.** The current default text, “Mathematical Thinking: Problem-Solving and Proofs,” was chosen because it has a large number of generally good exercises. In particular, it has exercise about logic which are fairly compelling. It also has a wide range of topics so it can accommodate different choices of emphasis. But it would be a mistake to try to cover a large portion of the book. Some find it helpful to pick a general topic or two for emphasis (like symbolic logic, or group theory), while others pick threads representing different subfields and develop the mathematics around these threads to the exclusion of wider theories.

2.3. **Some topics work better than others.** While in principle anything could be used, demands of subsequent study and experience in running the class has led us to favor the following topics.

- Elementary logic (optionally including symbolic logic).
- Sets and functions (for example, establishing de Morgan’s laws and that the preimage of an intersection is the intersection of the preimages).
- Divisibility (for example, establishing that a number is divisible by three if and only if the sum of its base-ten digits are).
- Sums of arithmetic and geometric sequences.
- Absolute values and inequalities (for example, a nice problem in the book is to describe when a quadratic polynomial is non-negative).
- Counting problems (for example, counting the subsets of a finite set).

2.4. **Supplementary materials might be needed.** The book is dense. If one is using it exclusively, expect discussion to help students read it appropriately. In the past, instructors have put on reserve a more friendly treatment of some topics covered, such as Berger and Starbird’s “The Heart of Mathematics” or photocopied out-of-print texts. Putting texts such as Polya’s on reserve could be helpful as well.